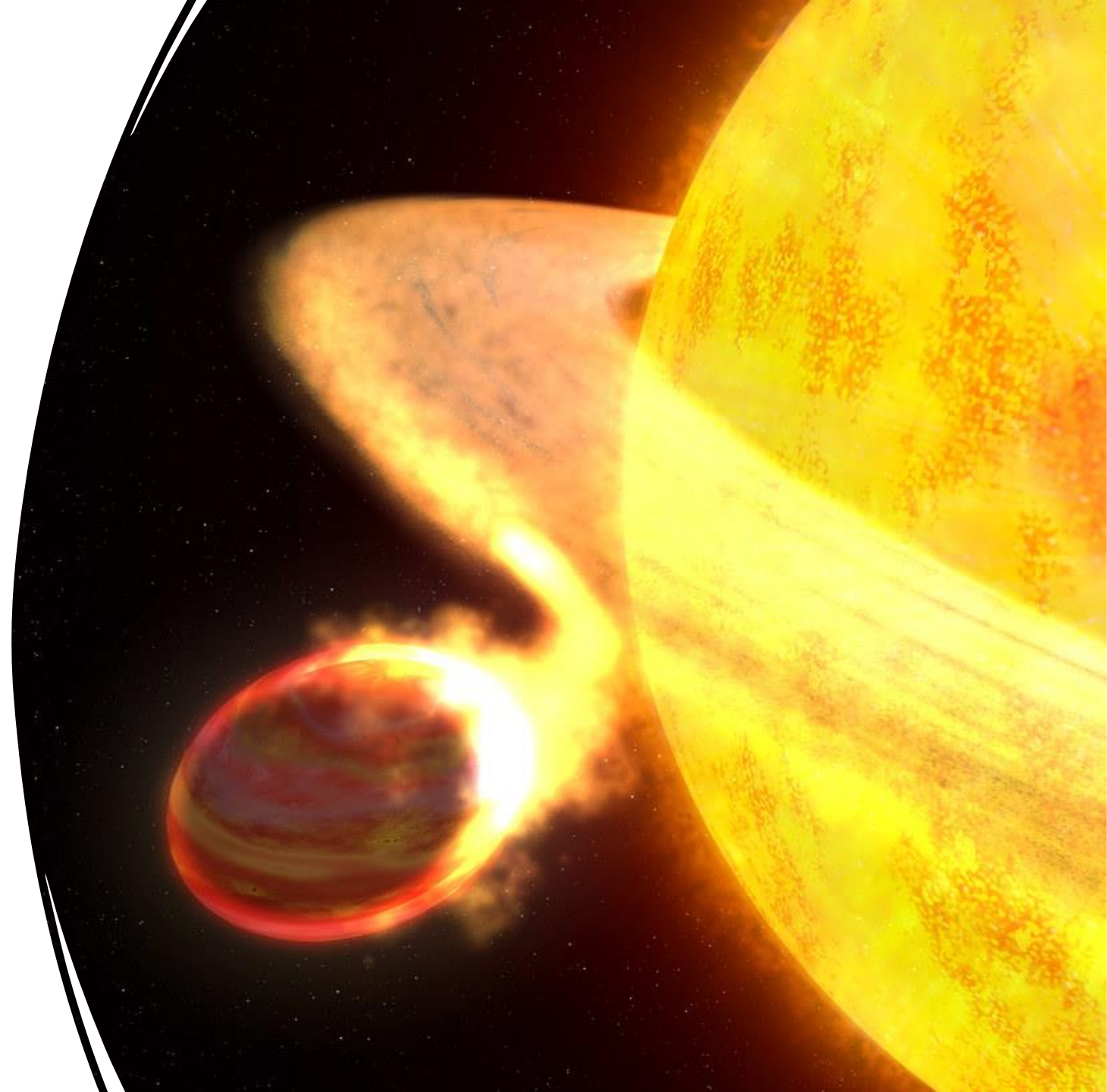


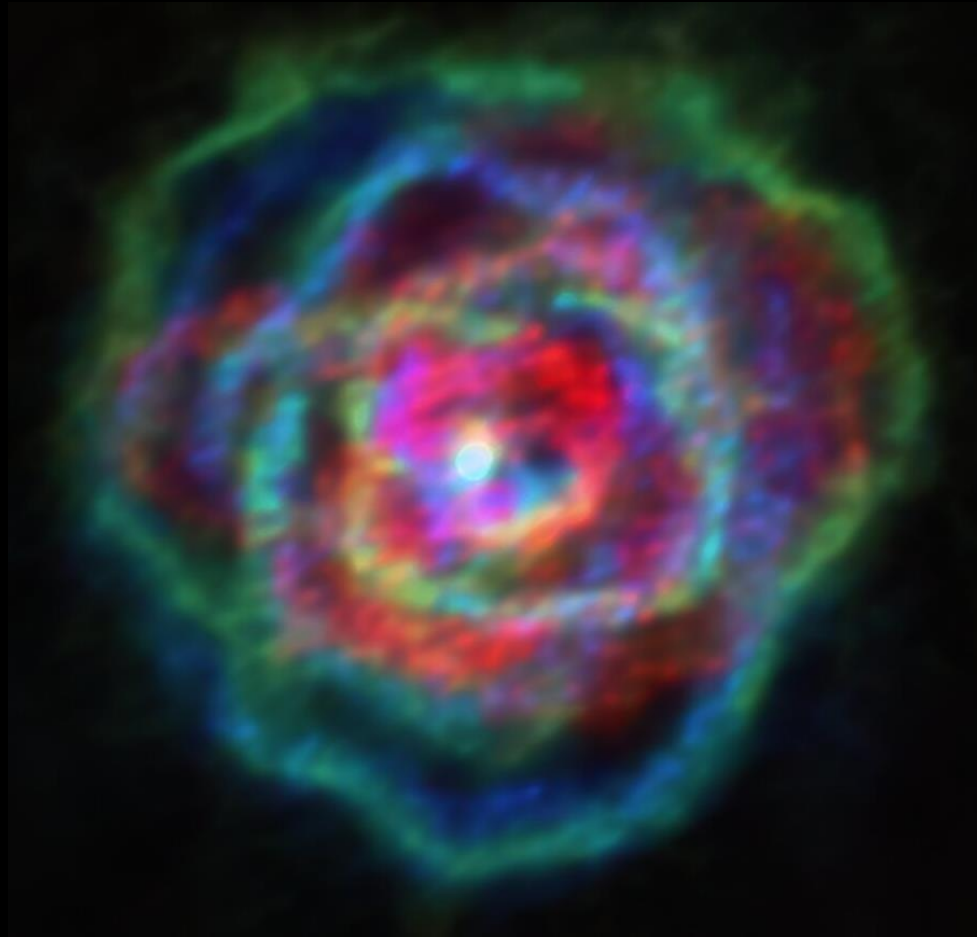
# Tidal Dissipation in Cool Evolved Stars

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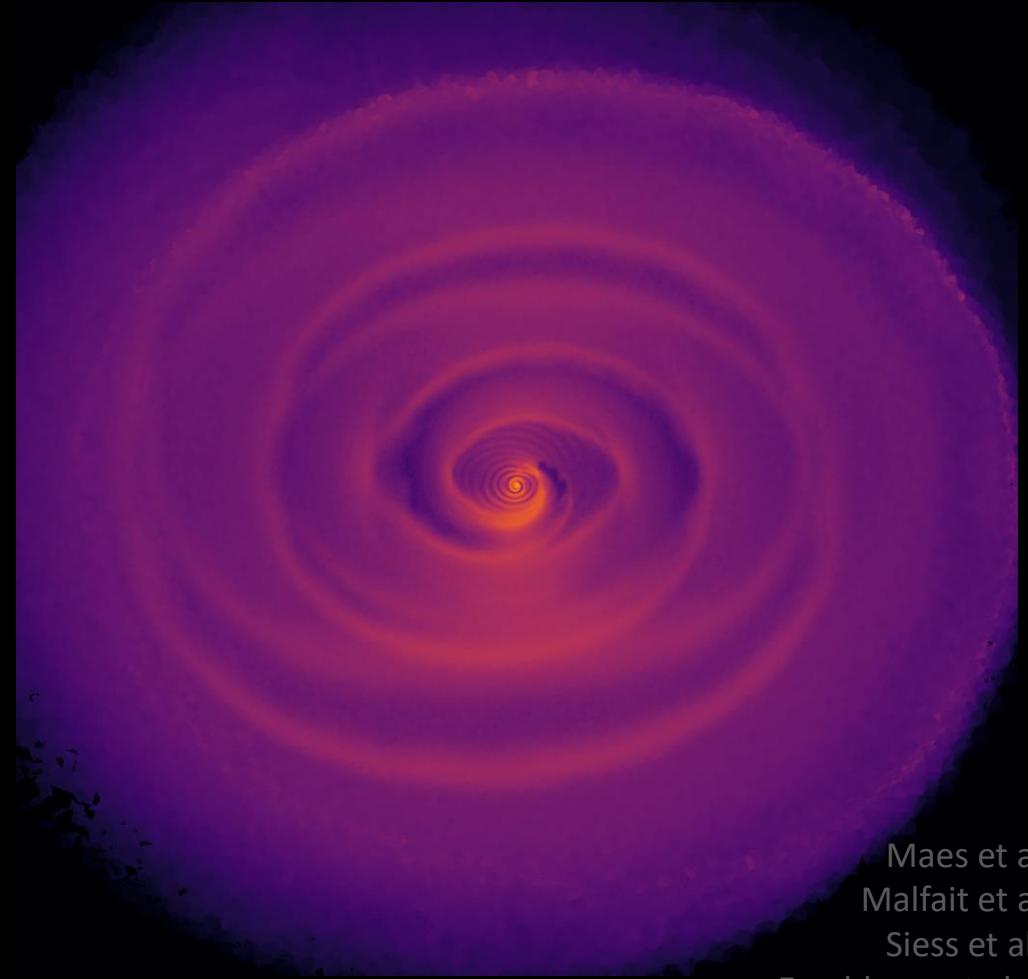
Mats Esseldeurs  
Stéphane Mathis  
Leen Decin



# Observations and Simulations



Decin et al. 2020

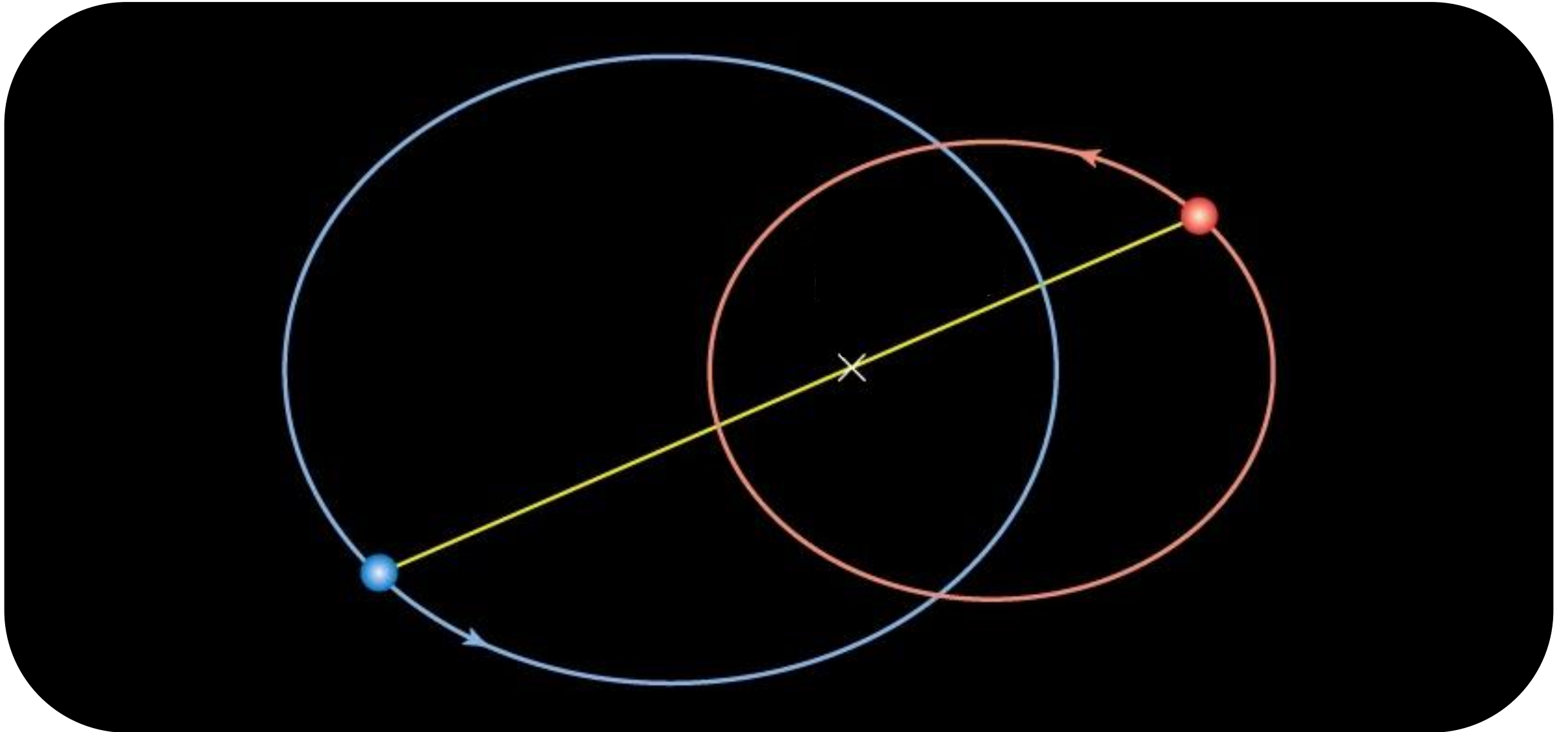


Maes et al. 2021  
Malfait et al. 2021  
Siess et al. 2022

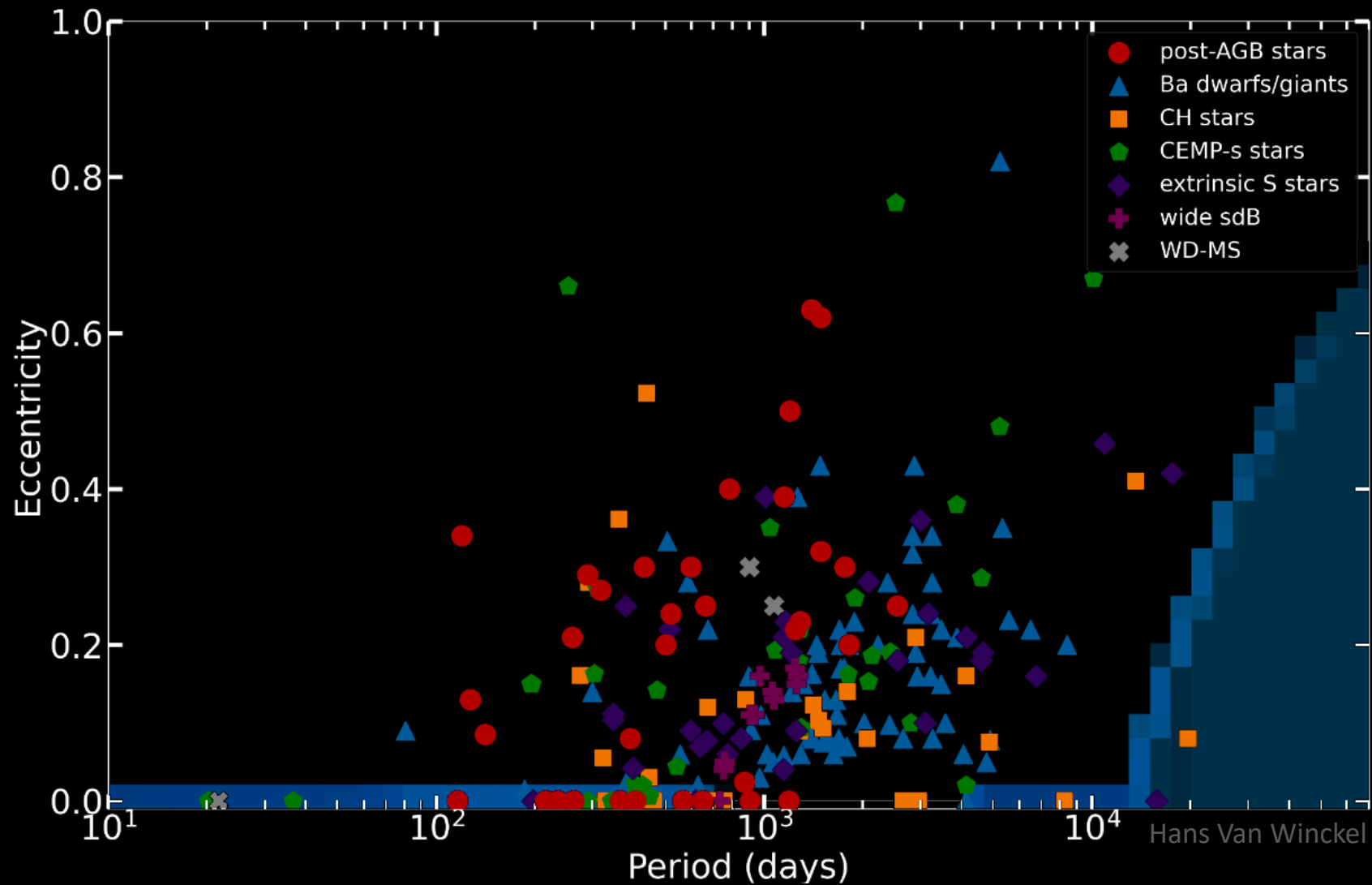
Esseldeurs et al. 2023

Malfait et al. 2024a,b (in prep)

# Orbital properties of binary systems

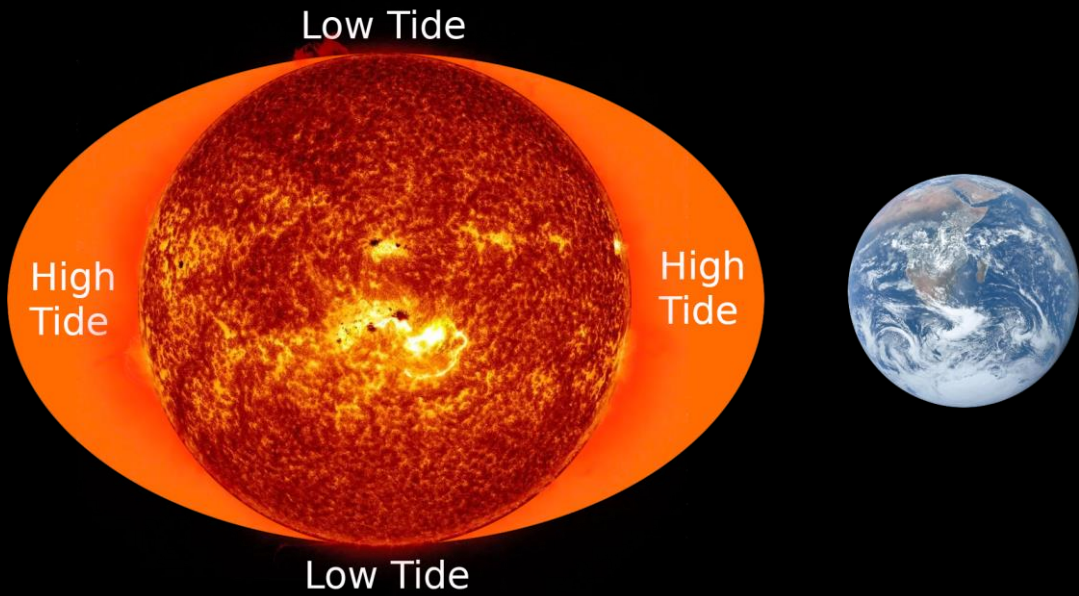


# Statistics of orbital properties



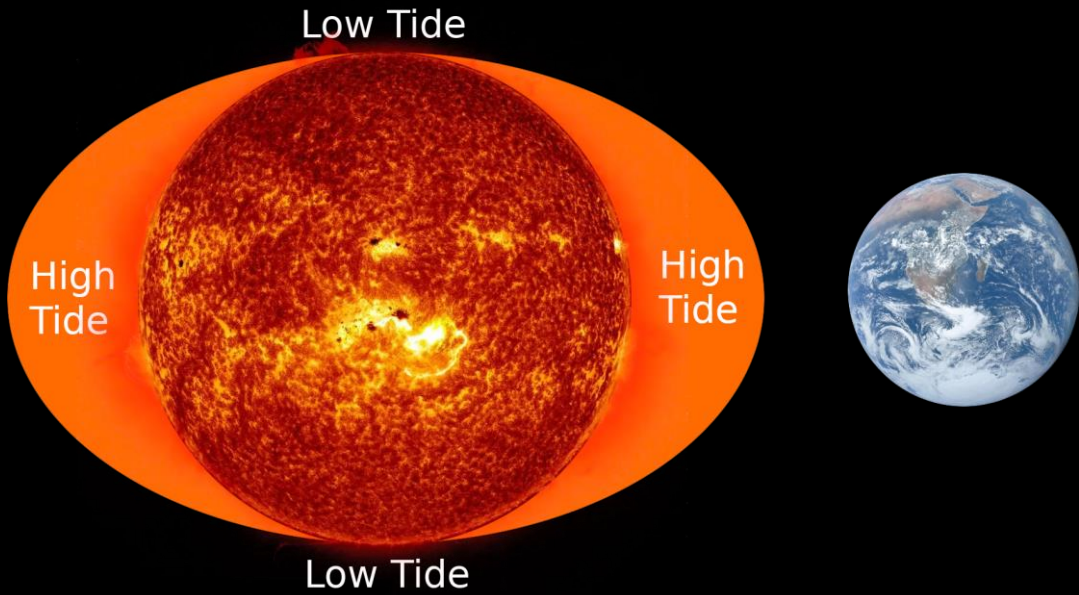
# Tidal Dissipation mechanisms

## Equilibrium Tides

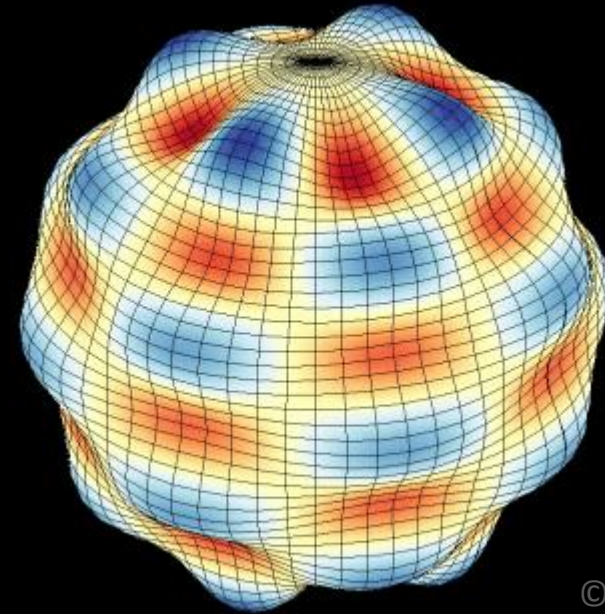


# Tidal Dissipation mechanisms

## Equilibrium Tides

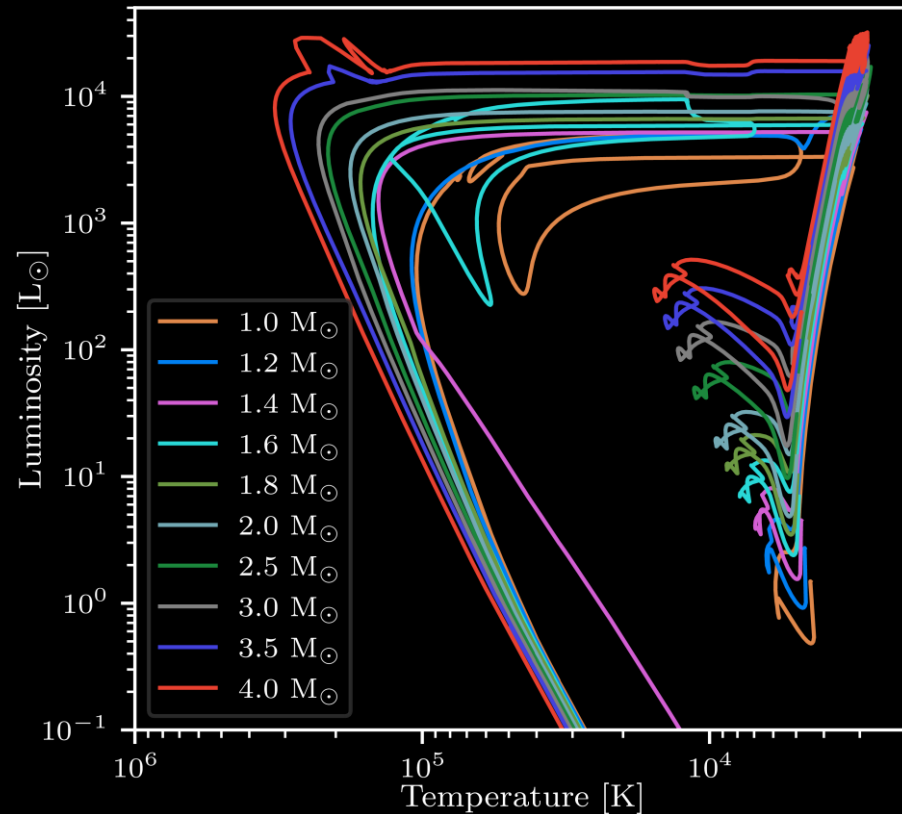


## Dynamical Tides



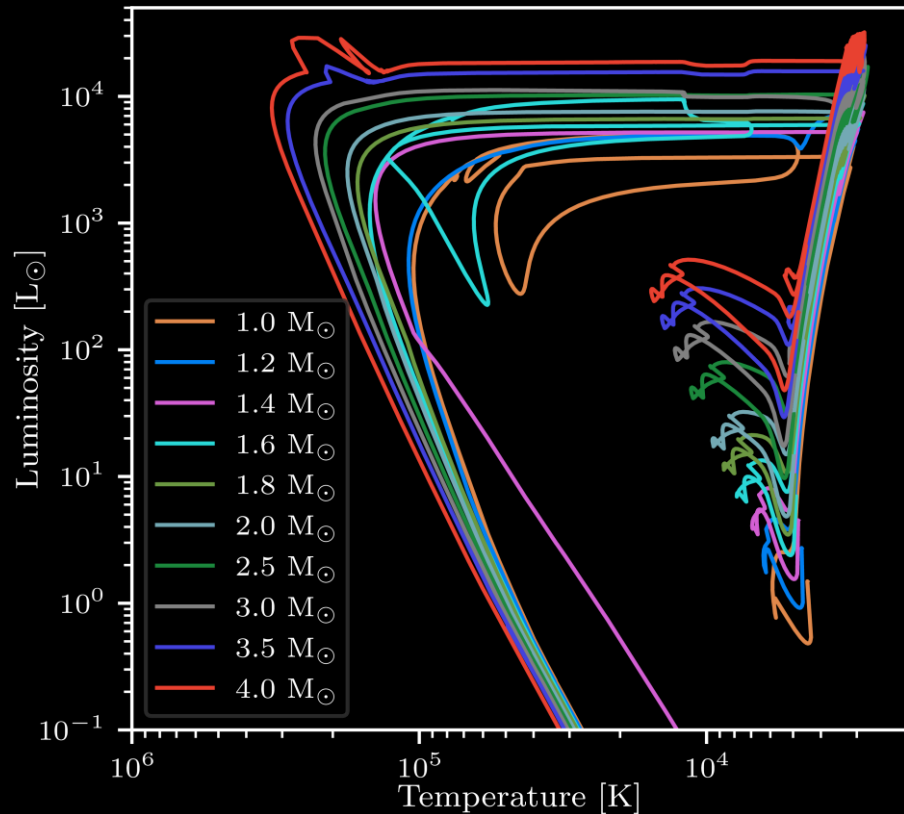
# Stellar Structure and Evolution

## Stellar evolution

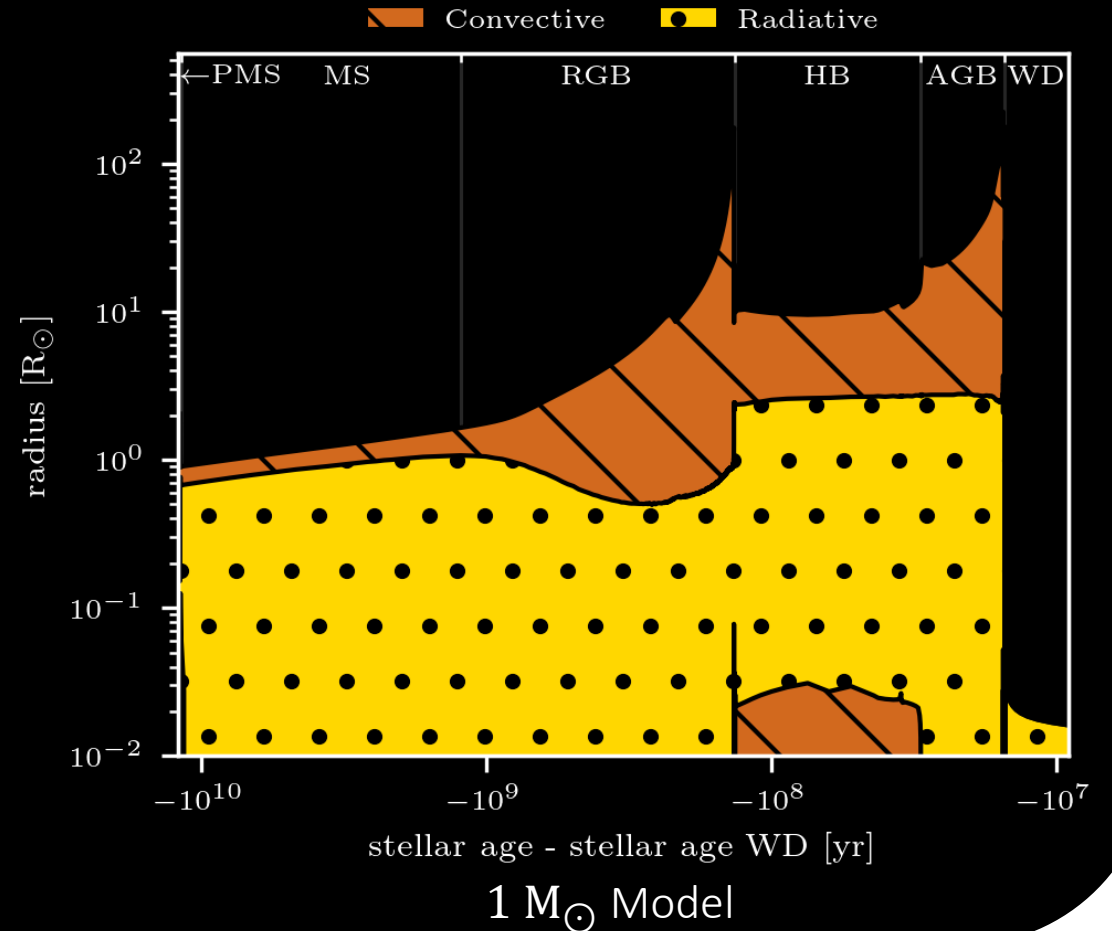


# Stellar Structure and Evolution

## Stellar evolution



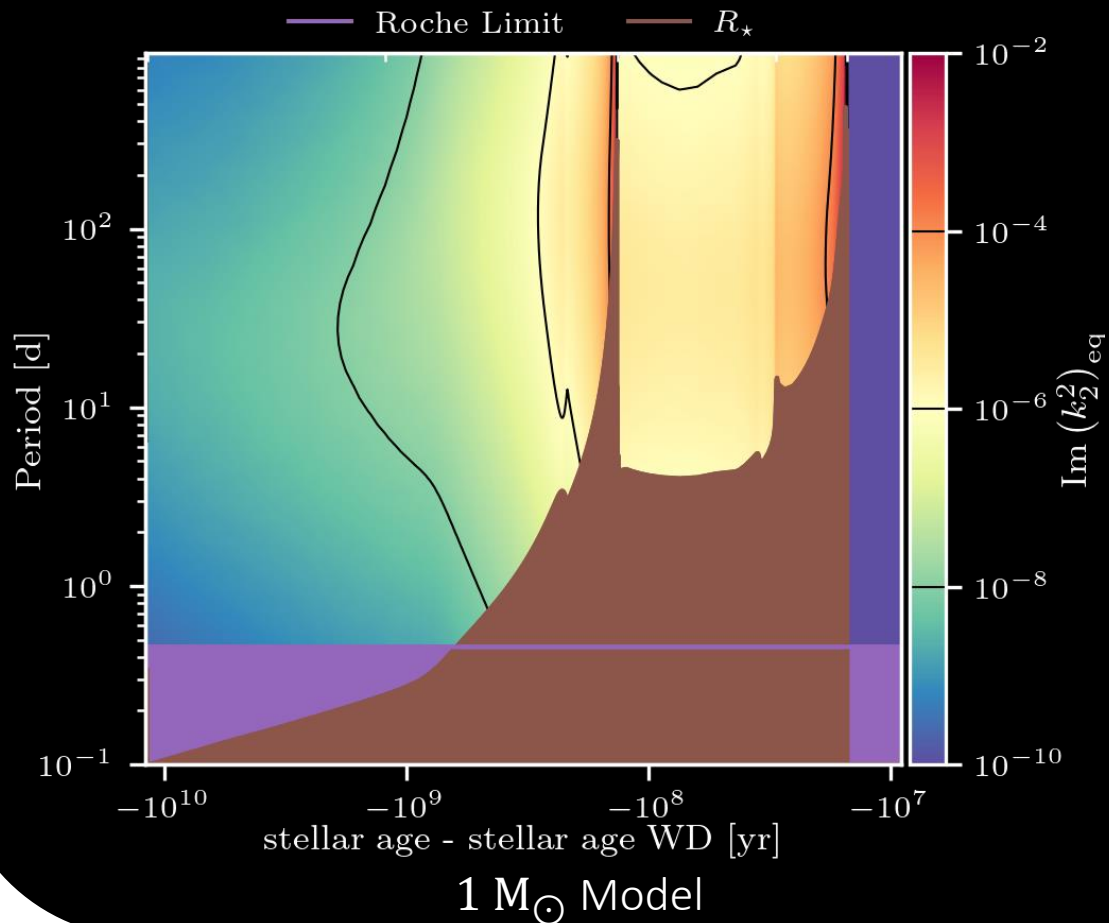
## Internal structure





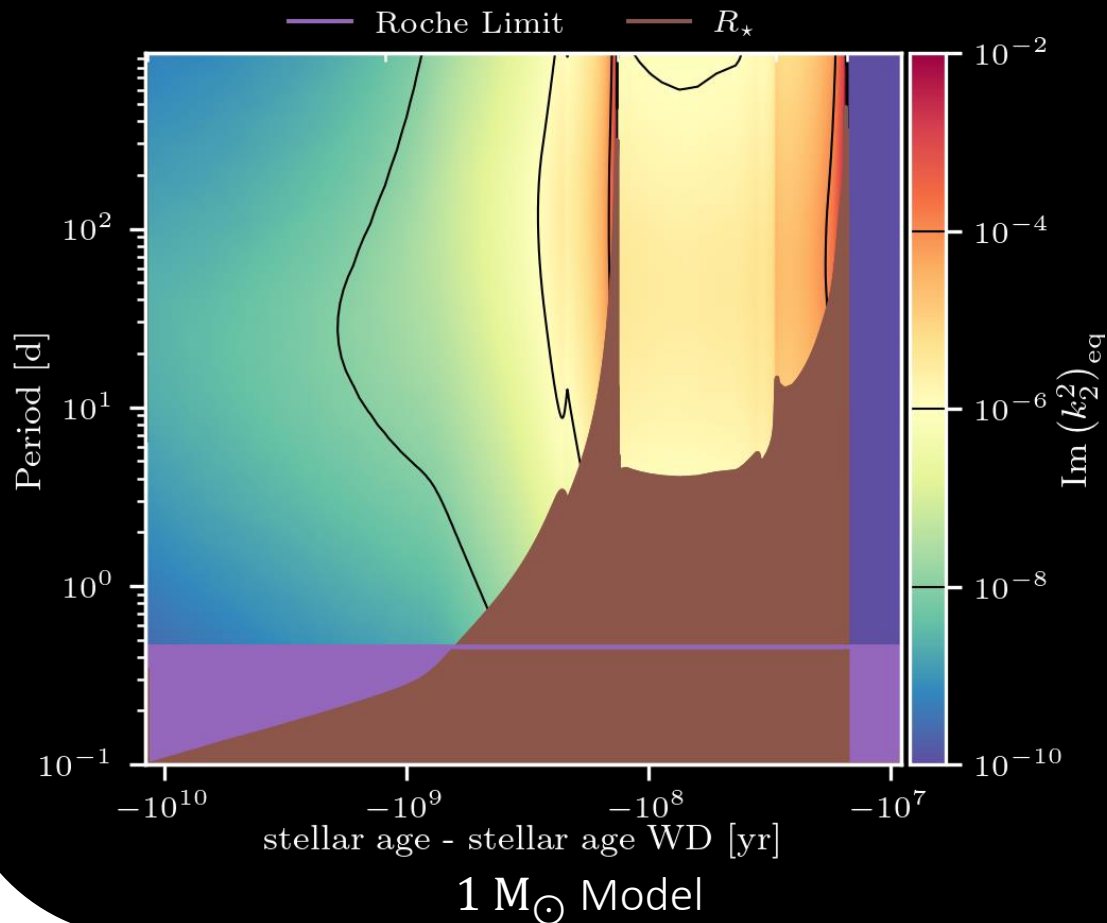
# Tidal Dissipation in Cool Evolved Stars

## Equilibrium Tides

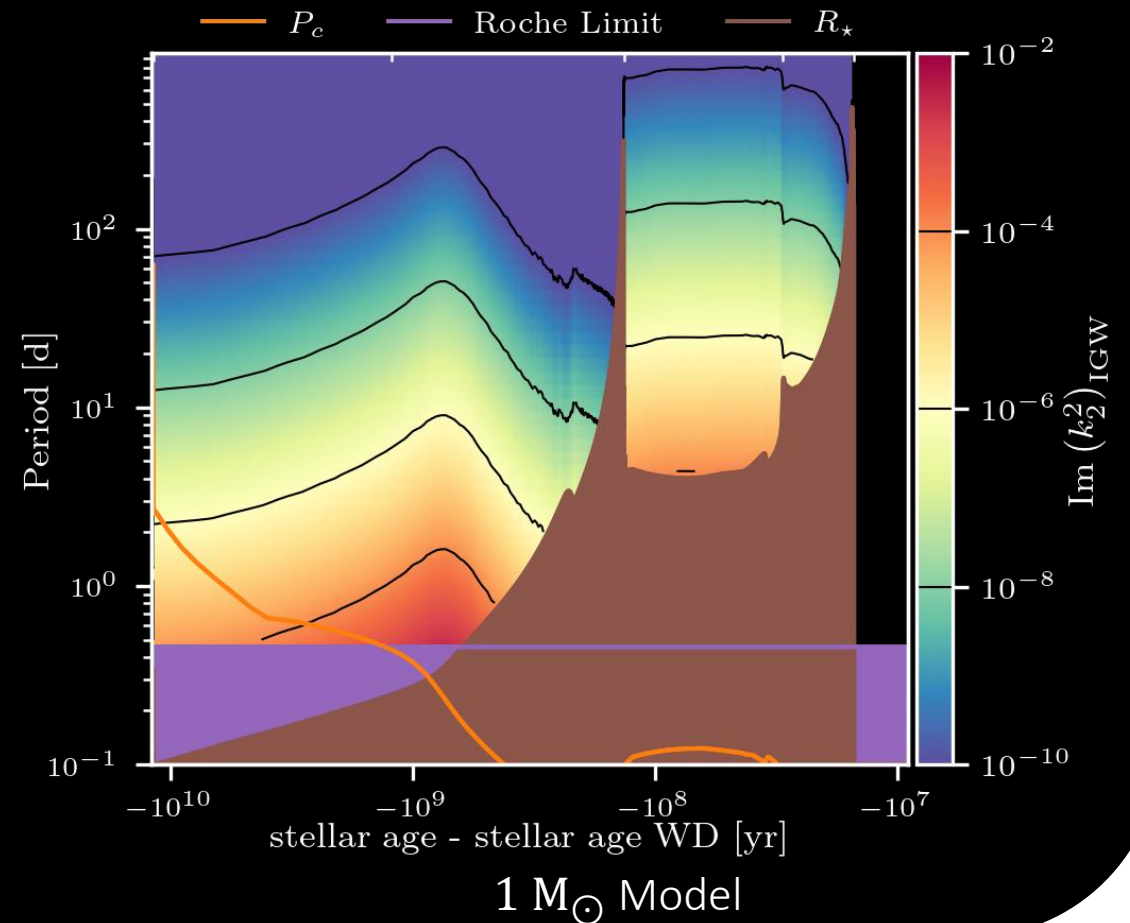


# Tidal Dissipation in Cool Evolved Stars

## Equilibrium Tides

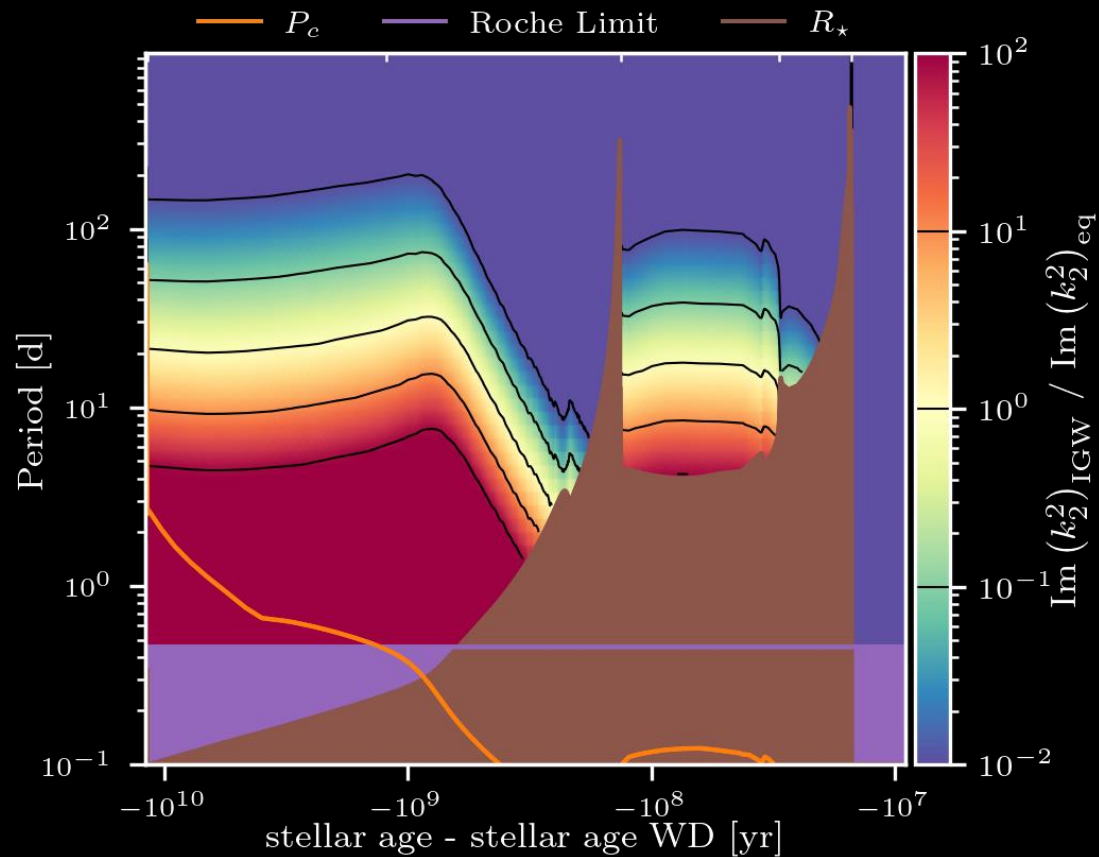


## Dynamical Tides



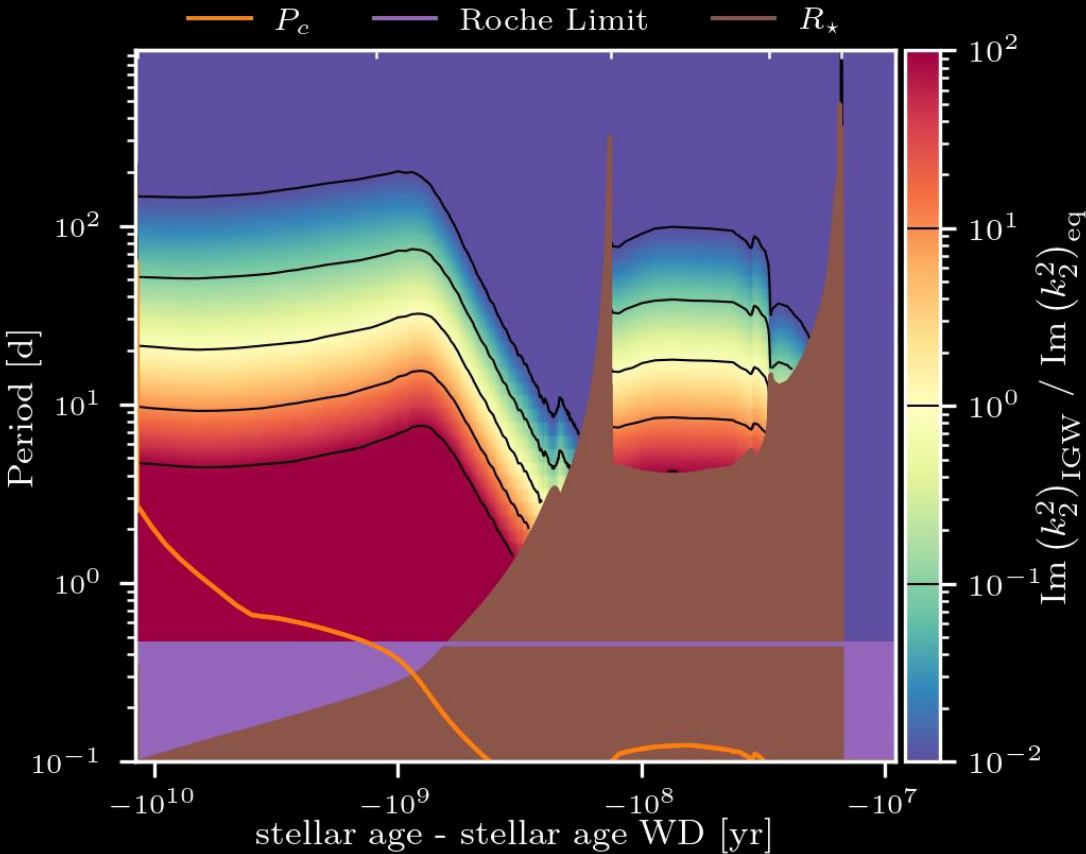
# Relative strengths of tidal dissipation

## 1 $M_{\odot}$ Model

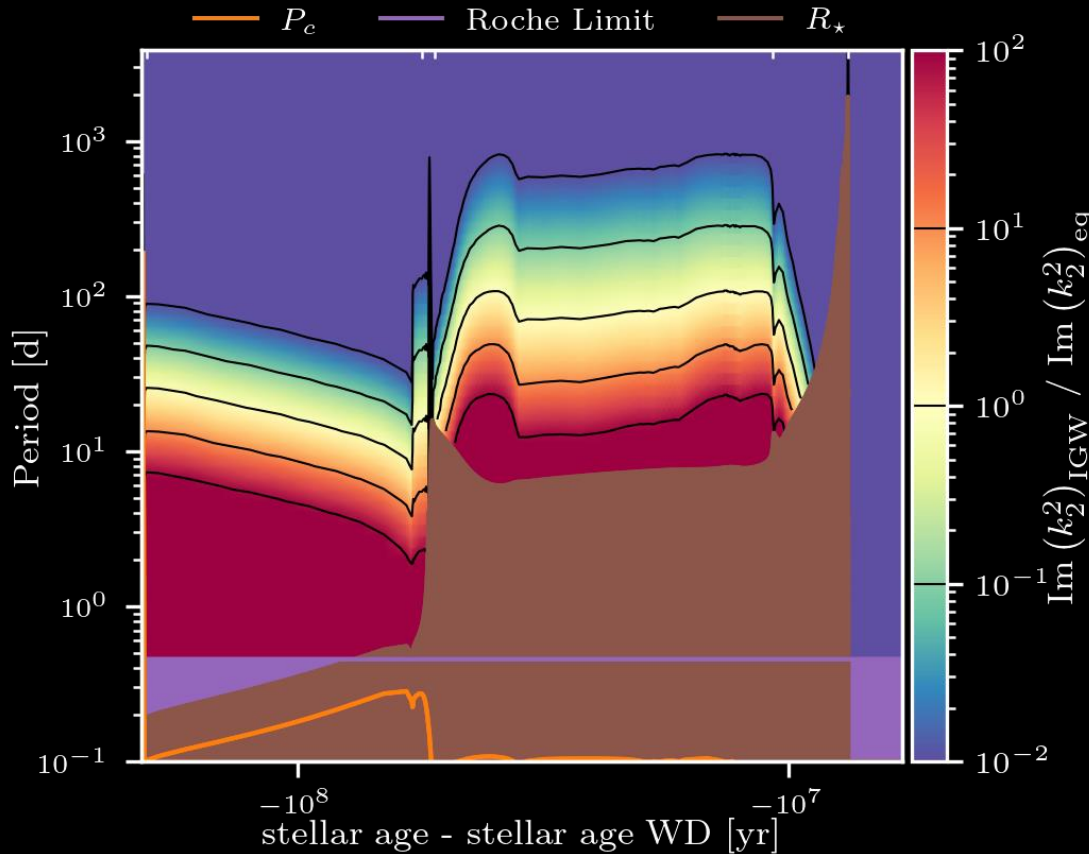


# Relative strengths of tidal dissipation

## 1 $M_{\odot}$ Model

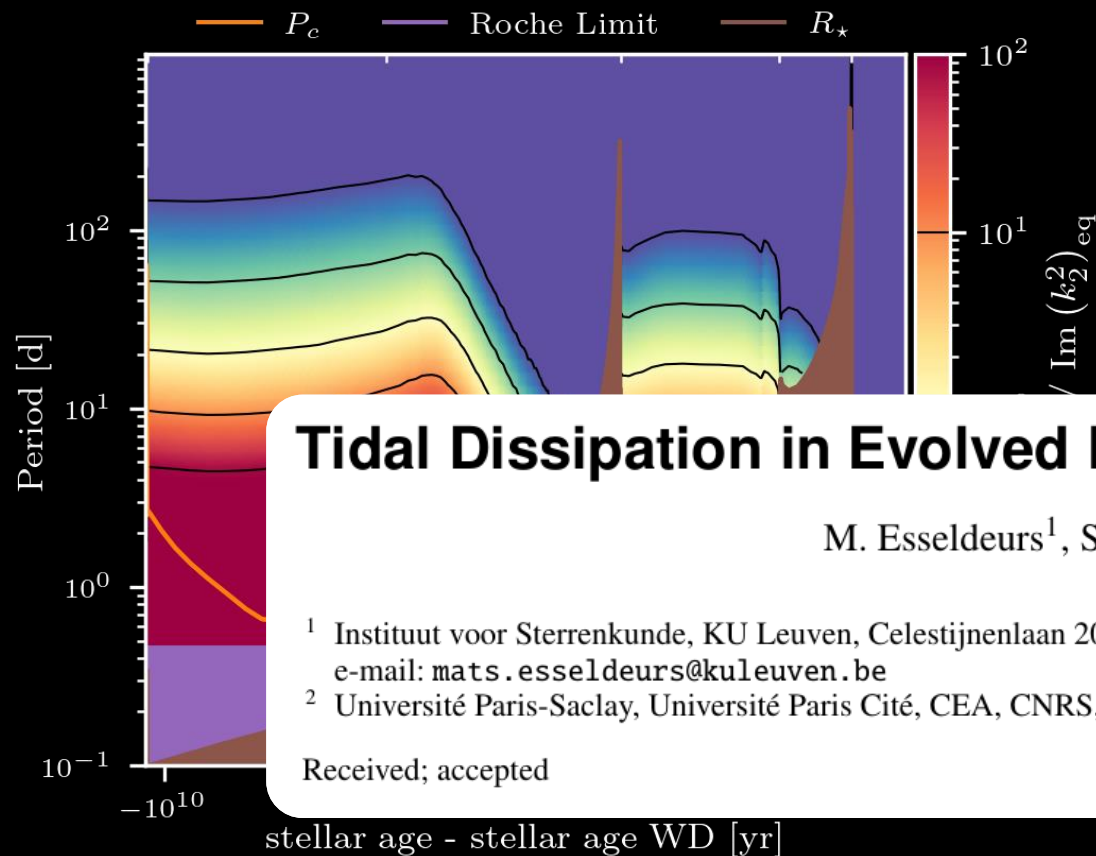


## 4 $M_{\odot}$ Model

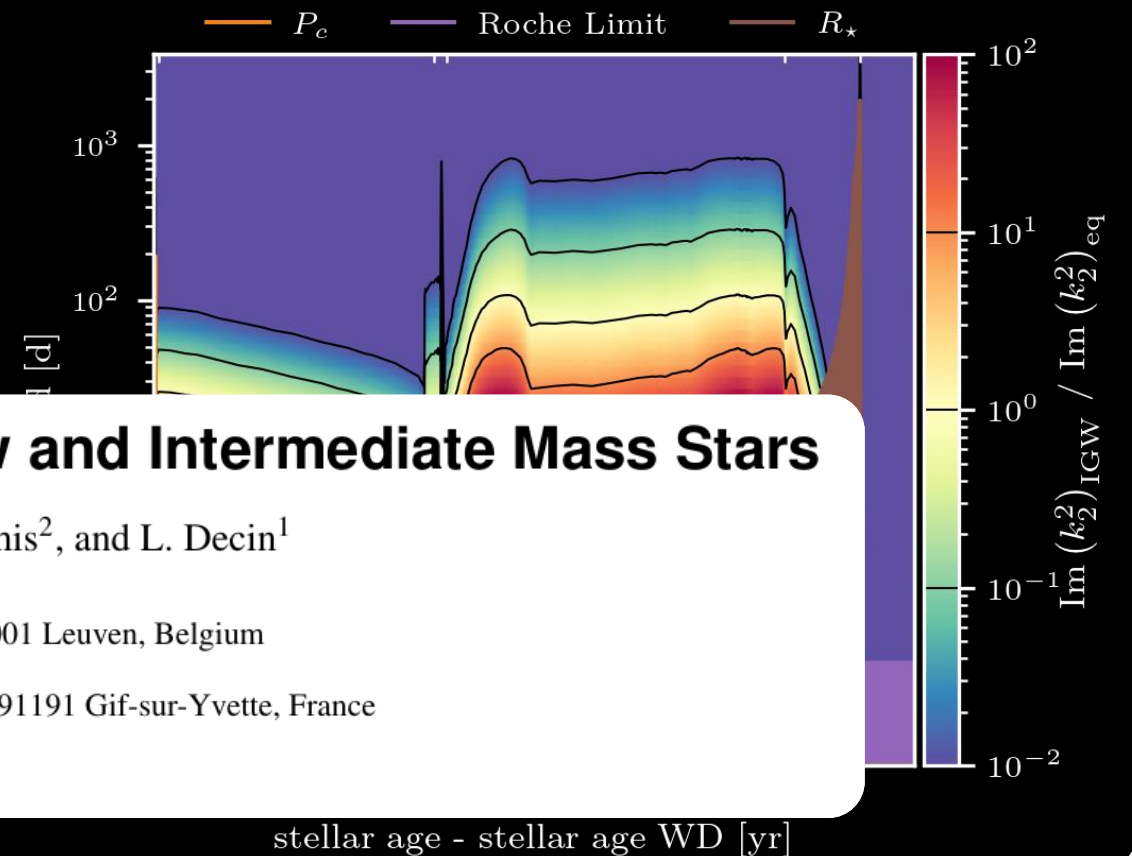


# Relative strengths of tidal dissipation

## 1 $M_{\odot}$ Model



## 4 $M_{\odot}$ Model



### Tidal Dissipation in Evolved Low and Intermediate Mass Stars

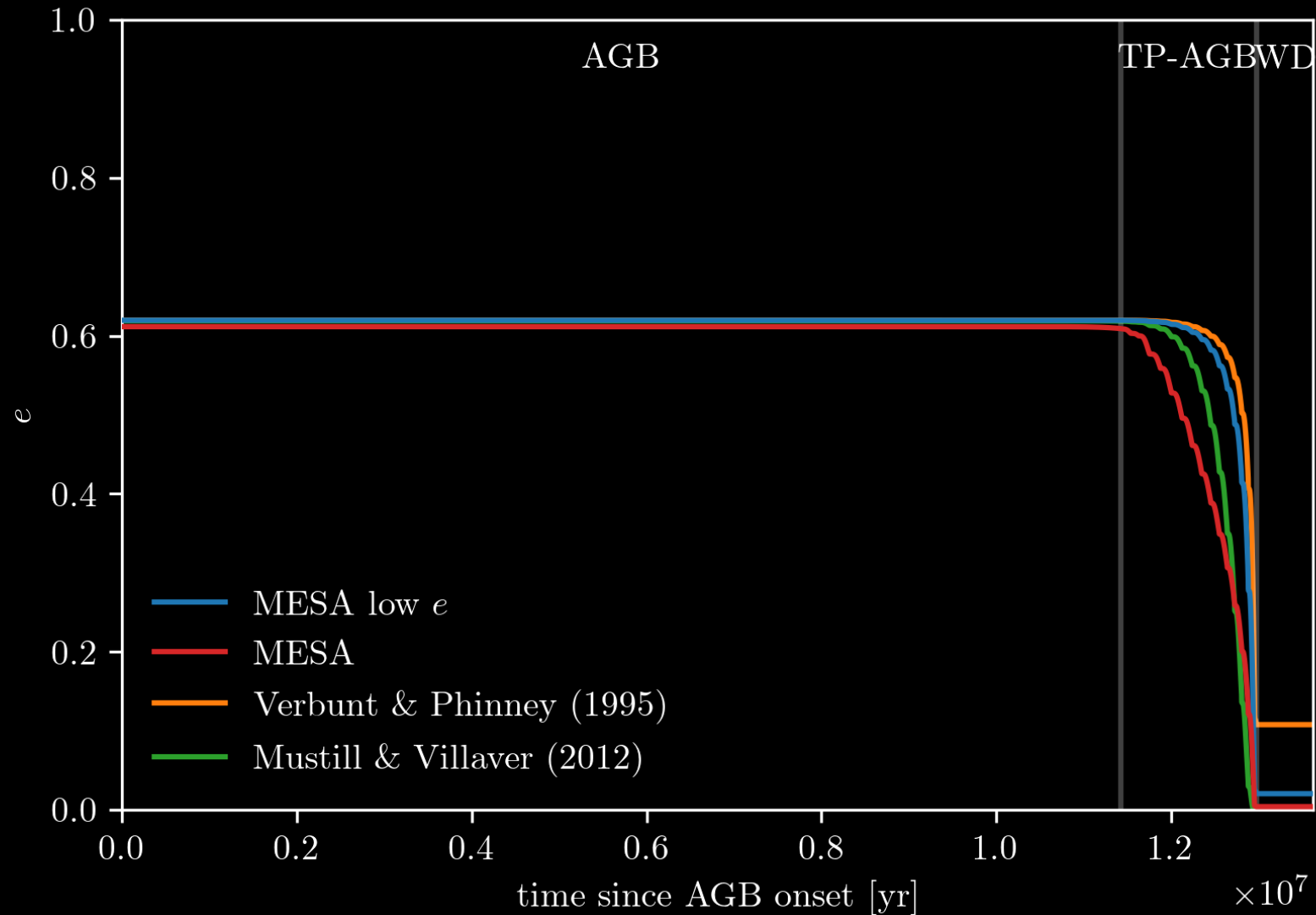
M. Esseldeurs<sup>1</sup>, S. Mathis<sup>2</sup>, and L. Decin<sup>1</sup>

<sup>1</sup> Instituut voor Sterrenkunde, KU Leuven, Celestijnenlaan 200D, 3001 Leuven, Belgium  
e-mail: mats.esseldeurs@kuleuven.be

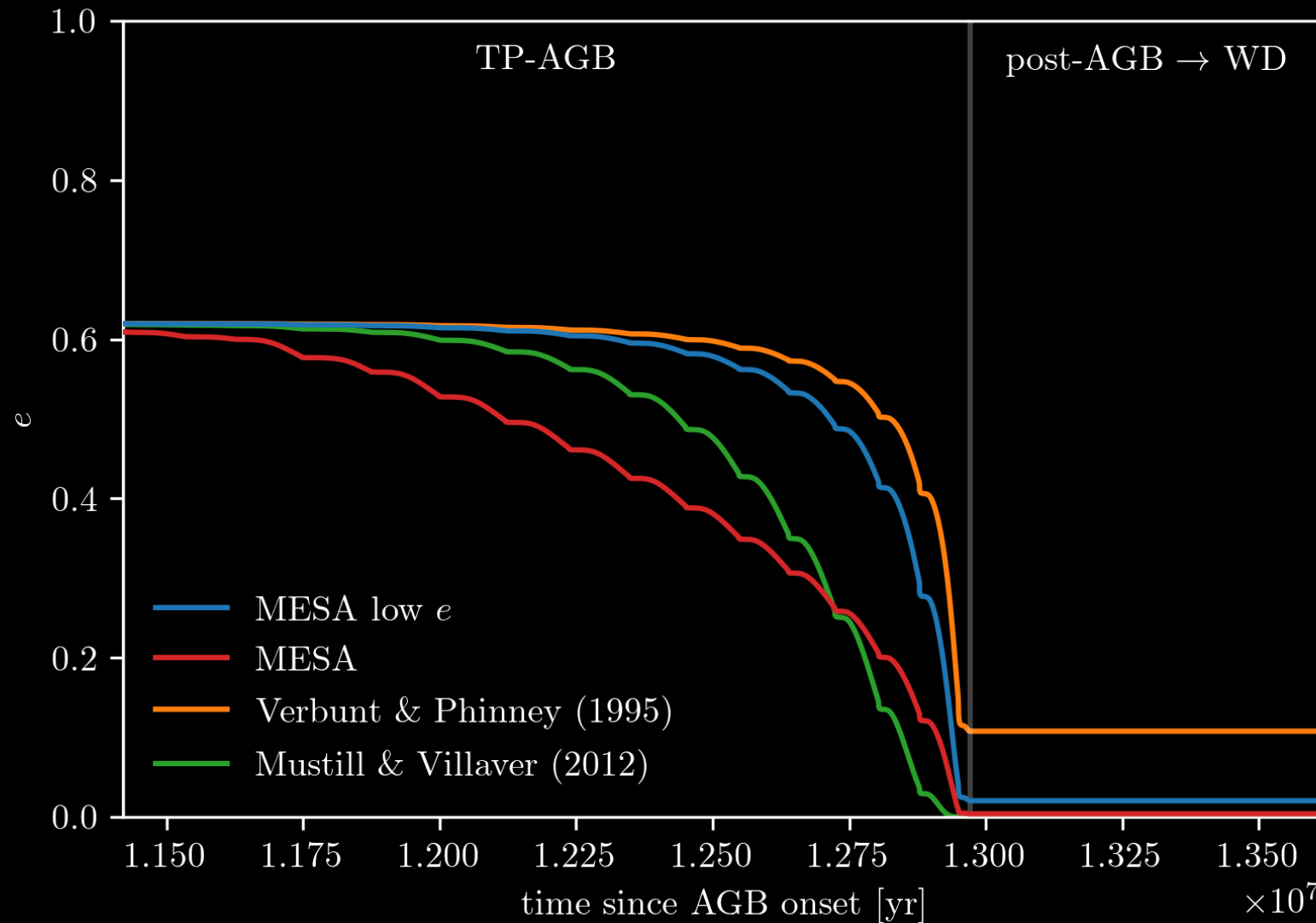
<sup>2</sup> Université Paris-Saclay, Université Paris Cité, CEA, CNRS, AIM, 91191 Gif-sur-Yvette, France

Received; accepted

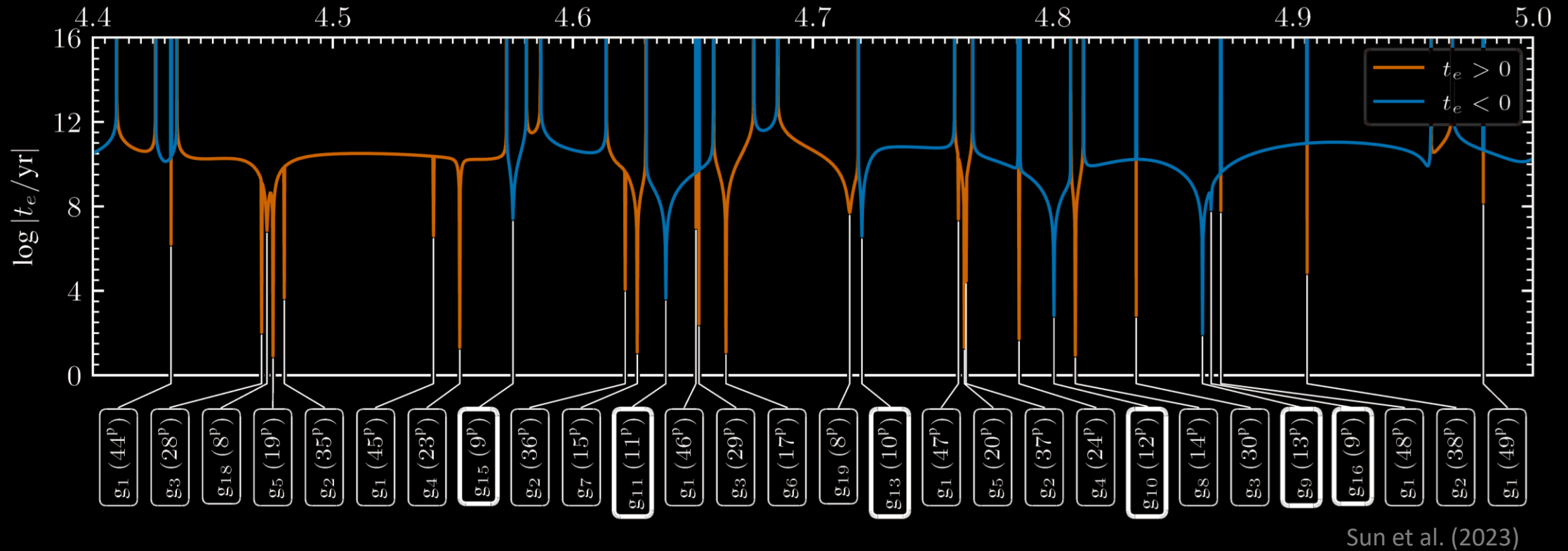
# Eccentricity change during the AGB phase



# Eccentricity change during the AGB phase



# Tidal eccentricity pumping through resonances

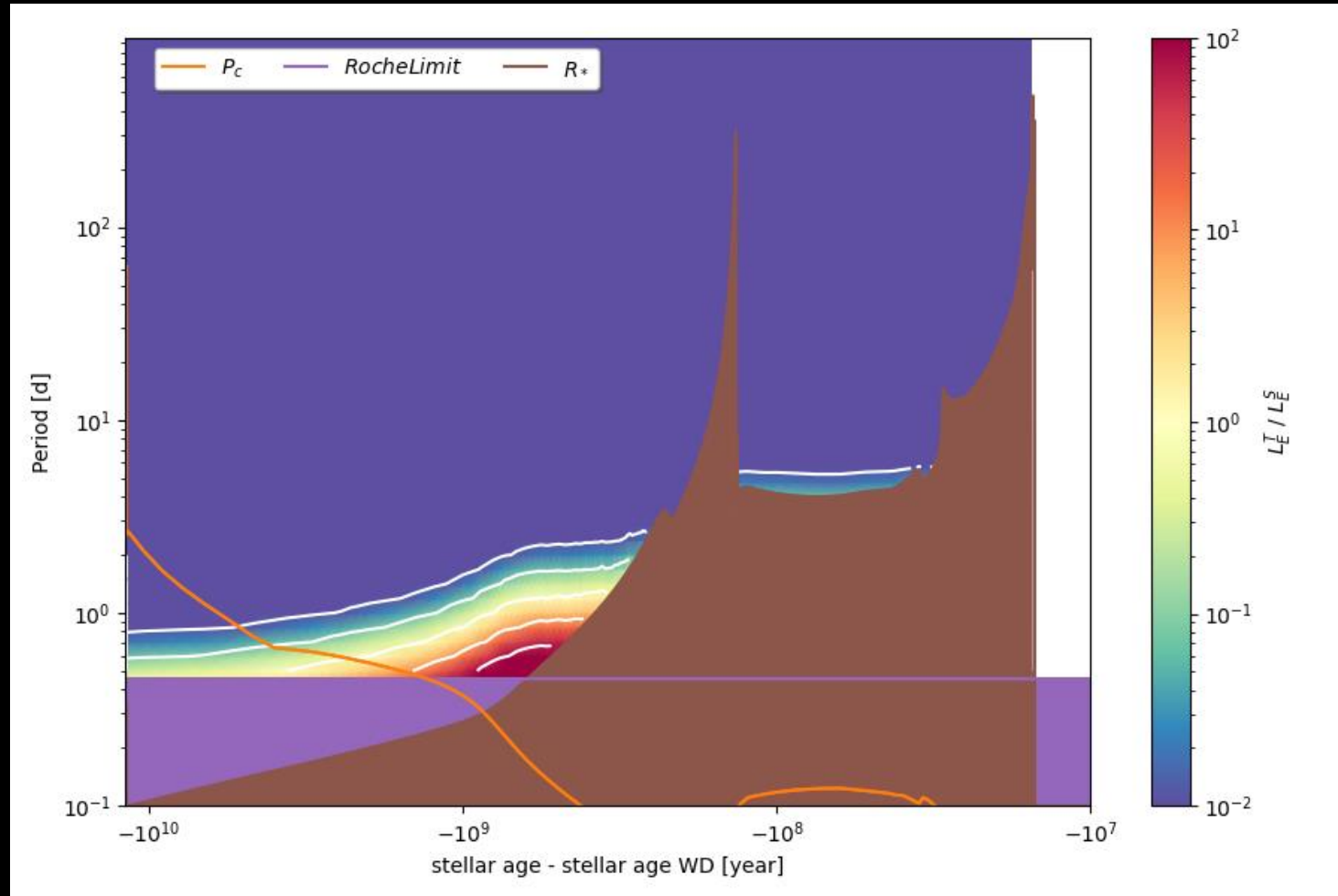




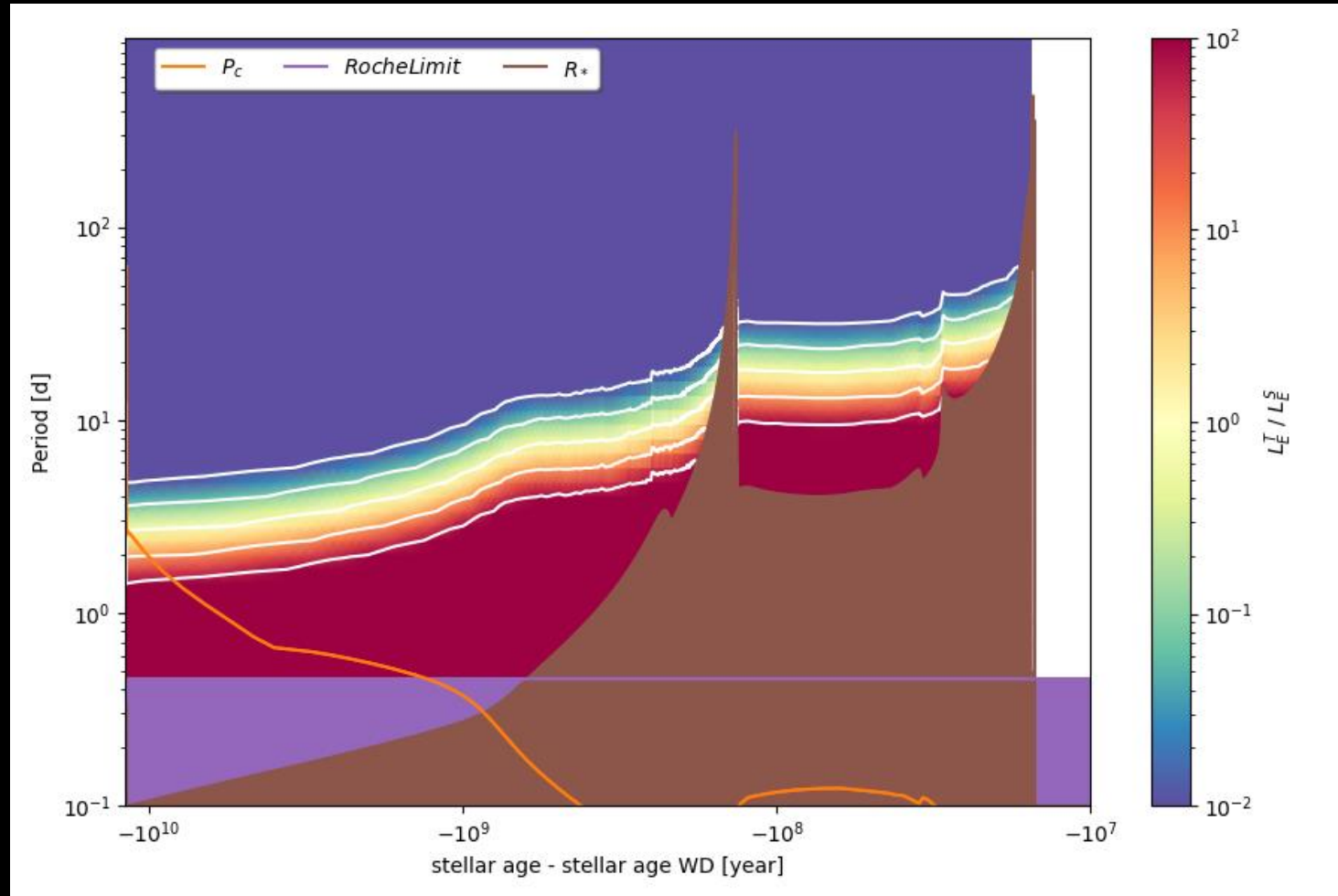
# Conclusions

- Tidal dissipation can be calculated ab-initio throughout the entire lifetime of a star
- The dynamical tide of gravity waves remains moderate during the giant phases
- The eccentricity problem is not yet solved
- The dynamical tide connecting with pressure modes remains to be studied

# Mixing due to tidal waves



# Mixing due to tidal waves



# Tidal Dissipation in Cool Evolved Stars

## Equilibrium Tides

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\Phi_l^{\text{nw}}}{dr} \right) - \frac{l(l+1)}{r^2} \Phi_l^{\text{nw}} - 4\pi G \frac{d\rho_0}{dr} \frac{1}{g_0} (\Phi_l^{\text{nw}} + \Psi_l) = 0$$

$$\begin{cases} \frac{d \ln \Phi_l^{\text{nw}}}{d \ln r} = l & \text{at } r = \eta R_\star \text{ for } \eta \rightarrow 0 \\ \frac{d \ln \Phi_l^{\text{nw}}}{d \ln r} = -(l+1) & \text{at } r = R_\star \end{cases}$$

$$\xi_{r,l}^{\text{nw}} = -\frac{\Phi_l^{\text{nw}} + \Psi_l}{g_0}, \quad \xi_{h,l} = \frac{1}{l(l+1)} \left( 2\xi_{r,l}^{\text{nw}} + r \frac{d\xi_{r,l}^{\text{nw}}}{dr} \right)$$

$$D_l(r) = \frac{1}{3} \left( 3 \frac{d\xi_{r,l}^{\text{nw}}}{dr} - \frac{1}{r^2} \frac{d(r^2 \xi_{r,l}^{\text{nw}})}{dr} + l(l+1) \frac{\xi_{h,l}^{\text{nw}}}{r} \right)^2 + l(l+1) \left( \frac{\xi_{r,l}^{\text{nw}}}{r} + r \frac{d(\xi_{h,l}^{\text{nw}}/r)}{dr} \right)^2 + (l-1)l(l+1)(l+2) \left( \frac{\xi_{h,l}^{\text{nw}}}{r} \right)^2,$$

$$\text{Im}(k_2^2)_{\text{eq}} = \frac{16\pi G \omega_t}{4(2l+1)R_\star |\varphi_T(R_\star)|^2} \int_0^{R_\star} r^2 \rho v_t D_l(r) dr,$$

## Dynamical Tides

$$\mathcal{F}_{\text{in}} = \int_0^{r_{\text{in}}} \left[ \left( \frac{r^2 \varphi_T}{g_0} \right)'' - \frac{l(l+1)}{r^2} \left( \frac{r^2 \varphi_T}{g_0} \right) \right] \frac{X_{1,\text{in}}}{X_{1,\text{in}}(r_{\text{in}})} dr$$

$$\mathcal{F}_{\text{out}} = \int_{r_{\text{out}}}^{R_\star} \left[ \left( \frac{r^2 \varphi_T}{g_0} \right)'' - \frac{l(l+1)}{r^2} \left( \frac{r^2 \varphi_T}{g_0} \right) \right] \frac{X_{1,\text{out}}}{X_{1,\text{out}}(r_{\text{out}})} dr,$$

$$\begin{cases} X_{1,\text{out}}'' - \frac{\partial_r \rho_0}{\rho_0} X_{1,\text{out}}' - \frac{l(l+1)}{r^2} X_{1,\text{out}} = 0 \\ X_{1,\text{out}}(r)_{r \rightarrow 0} \propto r^{1/2 + \sqrt{1/4 + l(l+1)}} \\ X_{1,\text{out}}'(r)_{r \rightarrow 0} \propto \left( 1/2 + \sqrt{1/4 + l(l+1)} \right) r^{-1/2 + \sqrt{1/4 + l(l+1)}} \\ X_{1,\text{in}}'' - \frac{\partial_r \rho_0}{\rho_0} X_{1,\text{in}}' - \frac{l(l+1)}{r^2} X_{1,\text{in}} = 0 \\ X_{1,\text{out}}(r)_{r \rightarrow R_\star} \propto \rho_0 \left( r - R_\star - \frac{\varphi_T(R_\star)}{g_0(R_\star)} \right) \\ X_{1,\text{out}}'(r)_{r \rightarrow R_\star} \propto \rho_0(R_\star) \end{cases}$$

# Tidal Dissipation in Cool Evolved Stars

## Equilibrium Tides

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\Phi_l^{\text{nw}}}{dr} \right) - \frac{l(l+1)}{r^2} \Phi_l^{\text{nw}} - 4\pi G \frac{d\rho_0}{dr} \frac{1}{g_0} (\Phi_l^{\text{nw}} + \Psi_l) = 0$$

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$$\mathcal{F}_{\text{in}} = \int_0^{r_{\text{in}}} \left[ \left( \frac{r^2 \varphi_T}{g_0} \right)'' - \frac{l(l+1)}{r^2} \left( \frac{r^2 \varphi_T}{g_0} \right) \right] \frac{X_{1,\text{in}}}{X_{1,\text{in}}(r_{\text{in}})} dr$$

$$\mathcal{F}_{\text{out}} = \int_{r_{\text{out}}}^{R_\star} \left[ \left( \frac{r^2 \varphi_T}{g_0} \right)'' - \frac{l(l+1)}{r^2} \left( \frac{r^2 \varphi_T}{g_0} \right) \right] \frac{X_{1,\text{out}}}{X_{1,\text{out}}(r_{\text{out}})} dr,$$

$$\text{Im}(k_2^2)_{\text{IGW}} = \frac{3^{-\frac{1}{3}} \Gamma^2\left(\frac{1}{3}\right)}{2\pi} [l(l+1)]^{-\frac{4}{3}} \omega_t^{\frac{8}{3}} \frac{a^6}{GM_2^2 R_\star^5}$$

$$\times \left( \rho_0(r_{\text{in}}) r_{\text{in}} \left| \frac{dN^2}{d \ln r} \right|_{r_{\text{in}}}^{-\frac{1}{3}} \mathcal{F}_{\text{in}}^2 + \rho_0(r_{\text{out}}) r_{\text{out}} \left| \frac{dN^2}{d \ln r} \right|_{r_{\text{out}}}^{-\frac{1}{3}} \mathcal{F}_{\text{out}}^2 \right)$$